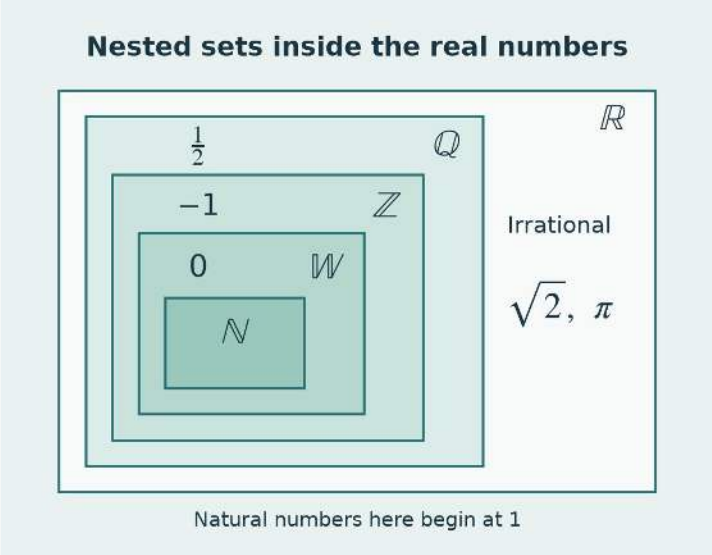
Number sets fit inside one another

$$\mathbb{N} \subset \mathbb{W} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$$

- Natural numbers here begin at 1; whole numbers also include 0.
- Integers add the negative whole numbers.
- An integer is also rational and real.



Classify the value, not its appearance

$$-12 = \frac{-12}{1}, \qquad \sqrt{81} = 9$$

- Simplify an expression before classifying it.
- A negative integer is rational, even without a fraction bar.
- A square-root symbol does not automatically mean irrational.

Simplify first, then classify

-12

Integer → rational → real

$0.375 = \frac{3}{8}$

Rational → real

$\sqrt{81} = 9$

Natural → whole → integer → rational → real

Rational means a ratio of integers

$$\frac{p}{q}, \quad p, q \in \mathbb{Z}, \quad q \neq 0$$

- A terminating decimal is rational.
- A repeating decimal is rational, even if it never ends.
- Irrational decimals neither terminate nor eventually repeat.

Ending, repeating or neither?

$0.375 = \frac{3}{8}$

Terminating: rational

$0.\overline{3} = \frac{1}{3}$

Repeating: rational

$\sqrt{2} = 1.41421356\ldots$

Neither ends nor repeats: irrational

Exact value and approximation are different

$$\sqrt{2} \approx 1.414, \qquad \sqrt{2} \neq 1.414$$

- $\sqrt{2}$ and π are exact irrational numbers.
- 1.414 and 3.14 are terminating rational decimals.
- Use $\approx$ for an approximation, not =.

The approximation is a different value

$\sqrt{2}$ irrational

$\approx$

$1.414 = \frac{707}{500}$

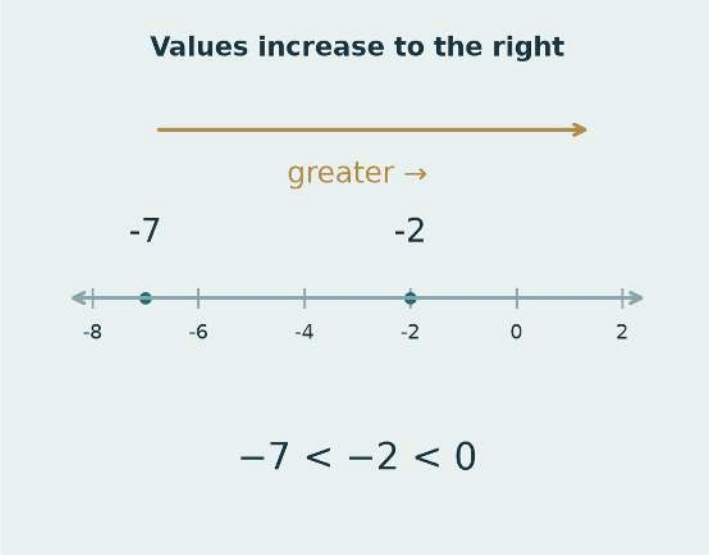
The written decimal is rational

Close to $\sqrt{2}$ does not mean equal to $\sqrt{2}$

Greater means farther right

$$-7 < -2 < 0$$

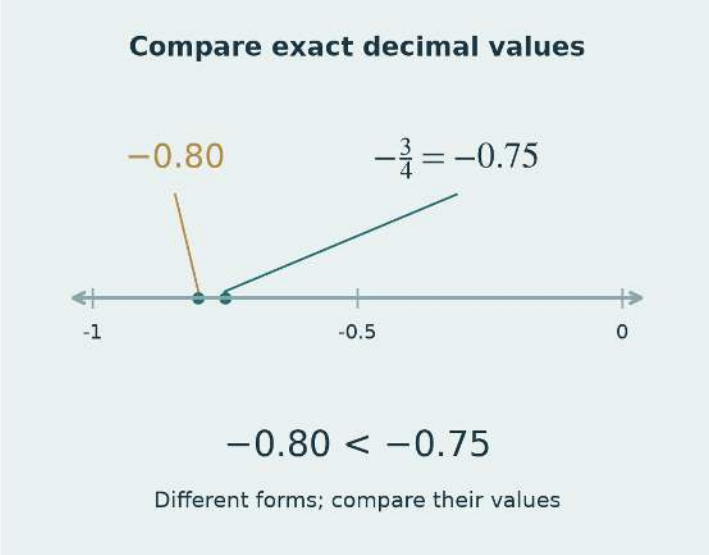
- Values increase as you move right.
- A negative value closer to zero is greater.
- Distance from zero and numerical order answer different questions.



Give fractions and decimals a common form

$$-\frac{3}{4} = -0.75 < -0.60 = -0.6$$

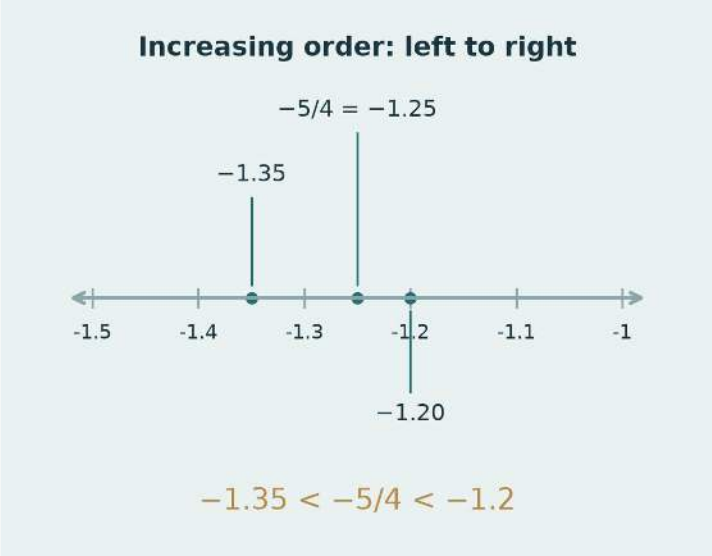
- Use an exact decimal or a common denominator.
- Keep negative signs attached to the values.
- Do not round away the difference you are comparing.



Order a mixed list without guessing

$$-1.35 < -\frac{5}{4} < -1.2$$

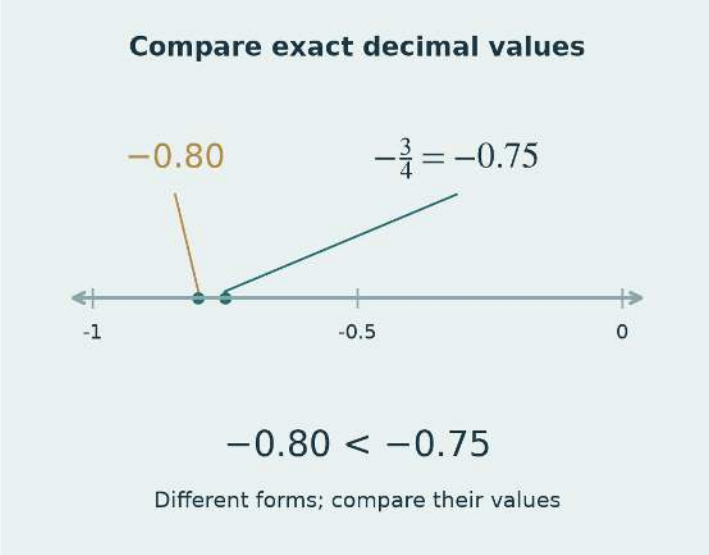
- Convert -5/4 exactly to -1.25.
- Compare -1.35, -1.25 and -1.20.
- Return to the original forms in your final list.



A scaled sketch can expose an error

$$-0.80 < -0.75 < 0$$

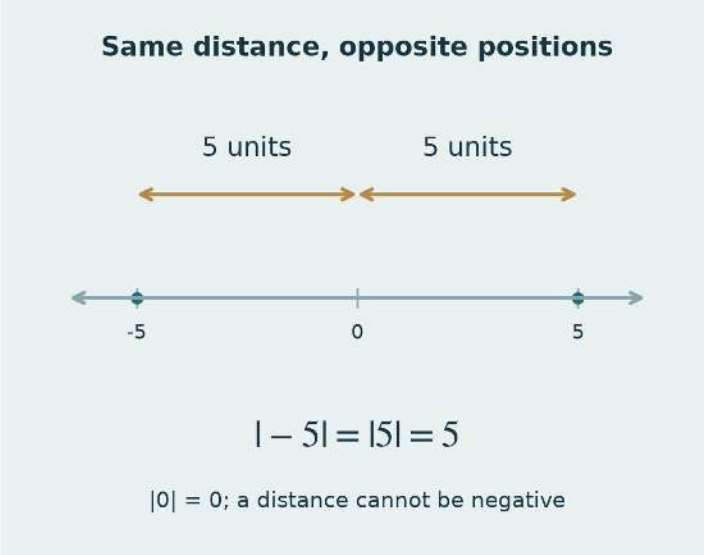
- Mark the origin and state the scale.
- Equivalent values occupy the same point.
- Explain a false claim with a numerical counterexample.



Opposites share a distance from zero

$$|-5| = |5| = 5, \qquad |0| = 0$$

- An opposite is the same distance away on the other side of 0.
- Absolute value gives that non-negative distance.
- Zero is its own opposite.



Evaluate inside the bars first

$$|-8|-|3-9| = 8-6 = 2$$

- The bars apply to the complete expression inside them.
- Work out 3-9 before taking its absolute value.
- An operation outside the bars stays outside.

Inside → absolute value → outside

$|-8| = 8$

$3-9 = -6$

$|3-9| = |-6| = 6$

$8-6 = 2$

Keep the subtraction outside the bars

Distance is not a signed change

$$\Delta = b - a, \qquad d = |b - a|$$

- From 5 to -3, the signed change is -8.
- The direct distance between the endpoints is 8.
- Reversing the direction changes the sign, not the distance.



Find all points at a stated distance

$$|x| = 4 \Rightarrow x = -4 \text{ or } x = 4$$

- A positive distance from zero gives two possible positions.
- Distance zero gives only the origin.
- Explain why a negative distance gives no real position.

