Solutions & reasoning

LESSON 1.1 / Geometry language and logical foundations

01 / Objects and notation

State how many endpoints a line, a ray and a segment have.

A line has 0, a ray 1, and a segment 2. The arrows indicate extension.

02 / Objects and notation

Name the ray with endpoint P that passes through Q, using standard geometric notation.

$\overrightarrow{PQ}$. Reversing to $\overrightarrow{QP}$ changes its endpoint.

03 / Objects and notation

Explain the difference between $\overline{AB}$ and $AB = 9\ \text{cm}$.

The first is a segment; the second states its numerical length is 9 cm.

04 / Objects and notation

Does reversing the letters change $\overline{AB}$ or $\overleftrightarrow{AB}$? Explain.

No. The endpoints or determining points are the same; segment AB is segment BA and line AB is line BA.

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05 / Objects and notation

Does $\overrightarrow{AB}$ equal $\overrightarrow{BA}$ for distinct A and B?

No. They have different endpoints and are different rays. They are not called opposite rays because opposite rays must share an endpoint.

06 / Objects and notation

A, B, C and D lie on a line in that order. Name the same ray as $\overrightarrow{AB}$ using D.

$\overrightarrow{AD}$; it starts at A and extends in the same direction.

07 / Position

For the order A-B-C-D, name a ray opposite to $\overrightarrow{BC}$.

$\overrightarrow{BA}$ is opposite to $\overrightarrow{BC}$.

08 / Position

A, B and C are collinear. Can they also be coplanar? Explain.

Yes. Any plane containing their line contains all three. The words describe different relationships.

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09 / Position

Three points are said to determine a unique plane. State the missing condition.

The three points must be noncollinear. Collinear points do not select one unique plane.

10 / Position

How many distinct lines pass through two distinct points P and Q? Name the accepted fact.

Exactly one. This is the two-distinct-points line postulate.

11 / Position

Can two distinct lines have two distinct common points? Justify.

No. The unique line through both common points would be both lines, contradicting that they are distinct.

12 / Position

Why is the finite shaded parallelogram used for a plane not a boundary of the mathematical plane?

A plane extends indefinitely. The finite shading is a visible window into the ideal surface, not a claim that the plane stops there.

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13 / Definitions and evidence

Classify: “A midpoint is a point on a segment dividing it into two congruent parts.”

Definition. Both the on-segment condition and the equal-parts condition are required.

14 / Definitions and evidence

Classify: “Exactly one line passes through two distinct points”, taken as an accepted starting fact.

Postulate or axiom. It is accepted as a starting statement in this geometric system.

15 / Definitions and evidence

A statement has been established from definitions, accepted starting facts and earlier results. What is its role?

It is a theorem. If it follows readily from an earlier theorem, it may be called a corollary.

16 / Definitions and evidence

A point X satisfies XA=XB but lies off segment AB. Is X its midpoint? Explain.

No. Equal distances do not replace the requirement that X lie on the segment.

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17 / Definitions and evidence

Two segments have matching single tick marks. What length equality is justified?

If the segments are AB and CD, $AB = CD$. The matching tick marks are the given evidence.

18 / Definitions and evidence

Two unmarked segments look equal. Is their equality false, proved, or not established by the sketch?

Not established by the sketch. They might be equal, but further evidence is needed.

19 / Reasoning

A learner measures five isosceles triangles and conjectures that every triangle is isosceles. Name the reasoning used to form the conjecture.

Inductive reasoning. The sample does not prove a statement about all triangles.

20 / Reasoning

Give a counterexample to “every triangle is isosceles”, explaining both conditions.

A 3-4-5 triangle is a triangle, yet its side lengths are all different. It meets the hypothesis and fails the conclusion.

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21 / Reasoning

Give a counterexample to “every quadrilateral with four equal sides is a square”.

A rhombus with four equal sides and a 60-degree interior angle. It has the required equal sides but fails the square’s right-angle condition.

22 / Reasoning

Write the converse of “if a quadrilateral is a square, then it is a rectangle”. Is the converse true?

If a quadrilateral is a rectangle, then it is a square. This is false: a 4-by-2 rectangle is a counterexample.

23 / Deduction

Given $AB = CD$, $CD = EF$ and $EF = 12\ \text{cm}$, determine AB. Name a reason.

$AB = 12\ \text{cm}$. Transitivity gives AB=EF; substitution supplies 12 cm.

24 / Challenge

M is stated to be the midpoint of segment AB. Given $AM = 3x + 1$ and $MB = 5x - 7$, find x and AB. State the reasons.

$3x + 1 = 5x - 7 \Rightarrow x = 4$. Each half has length 13, so AB=26 by segment addition. The midpoint is given, not inferred from appearance.