Solutions & reasoning

1.1 Functions, notation, domain & range

01 / Fluency

Is {(-2,3), (0,3), (2,7)} a function? Explain.

Yes. Each of -2, 0 and 2 has exactly one output. Repeating output 3 is allowed.

02 / Fluency

Is {(1,2), (1,-2), (3,4)} a function? Explain.

No. Input 1 is paired with both 2 and -2.

03 / Fluency

For $f(x)=2x^2-3x+1$, find $f(-3)$.

$f(-3)=2(9)+9+1=28$.

04 / Fluency

For $g(t)=3t^2-2$, simplify $g(a-2)$.

$g(a-2)=3(a^2-4a+4)-2=3a^2-12a+10$.

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05 / Fluency

For $f(x)=x^2+2x$, simplify $f(x+h)$.

$(x+h)^2+2(x+h)=x^2+2xh+h^2+2x+2h$.

06 / Fluency

For $f(x)=2x^2+1$, solve $f(x)=19$.

$2x^2+1=19$ gives $x^2=9$, so $x=-3$ or $x=3$.

07 / Domains

Find the largest real domain of $5/(x-7)$.

$x\neq 7$. Domain: $(-\infty,7) \cup (7,\infty)$.

08 / Domains

Find the largest real domain of $\sqrt{(9-x)}$.

$x\leq 9$. Domain: $(-\infty,9]$. The endpoint is allowed because $\sqrt{0}=0$.

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09 / Domains

Find the largest real domain of $(x+1)/(x^2-9)$.

$(x-3)(x+3)\neq 0$, so $x\neq -3$ and $x\neq 3$. Domain: $(-\infty,-3) \cup (-3,3) \cup (3,\infty)$.

10 / Domains

Find the largest real domain of $\sqrt{(2x+6)}/(x-4)$.

$2x+6\geq 0$ gives $x\geq -3$, and $x\neq 4$. Domain: $[-3,4) \cup (4,\infty)$.

11 / Domains

Find the largest real domain of $1/\sqrt{(2x-6)}$.

$2x-6>0$ gives $x>3$. Domain: $(3,\infty)$.

12 / Domains

Find the domain of $\sqrt{(5-x)}/(x+2)$.

$x\leq 5$ and $x\neq -2$. Domain: $(-\infty,-2) \cup (-2,5]$.

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13 / Ranges

Find the domain and range of $(x+3)^2-5$ on its largest real domain.

Domain: $\mathbb{R}$. The square is ≥0, with equality at x=-3, so the range is $[-5,\infty)$. Every y≥-5 occurs at $x=-3+\sqrt{(y+5)}$.

14 / Ranges

Find the domain and range of $\sqrt{(x-5)}+2$.

Domain: $[5,\infty)$. Range: $[2,\infty)$, since $\sqrt{(x-5)}$ attains every non-negative number.

15 / Ranges

Find the range of $(x-2)^2-4$ for $0\le x\le 3$.

q(0)=0, q(3)=-3 and q(2)=-4. The continuous curve attains all outputs from -4 to 0. Range: [-4,0].

16 / Ranges

Find the range of $(x+2)^2+1$ for $-4\le x\le 1$.

The minimum is 1 at x=-2. Endpoint values are 5 and 10. The maximum is 10, so range: [1,10].

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17 / Ranges

Find the range of $3-(x-1)^2$ for $-1 \le x \le 4$.

The maximum is 3 at x=1. Endpoint values are -1 and -6. Range: [-6,3].

18 / Ranges

Find the largest real domain and range of $2+1/(x-3)$.

Domain: $\mathbb{R}$ except 3. Range: $\mathbb{R}$ except 2. For $y \neq 2$, $x=3+1/(y-2)$ is a valid input that gives y.

19 / Model and justify

Tickets cost $15 and capacity is 60. Let R(n)=15n. State the contextual domain and range.

Domain: {0,1,2,…,60} tickets. Range: {0,15,30,…,900} dollars. Intervals would incorrectly include fractional tickets.

20 / Model and justify

A tank starts with 120 liters and drains at 3 liters/minute. Model it until empty and state domain and range.

V(t)=120-3t. Domain: [0,40] minutes; range: [0,120] liters. The model stops at t=40.

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21 / Model and justify

Bike hire costs C(h)=15+8h for any duration from 0.5 to 6 hours. State domain and range with units.

Domain: [0.5,6] hours. C(0.5)=19 and C(6)=63; range: [19,63] dollars.

22 / Model and justify

A student cancels x-3 in $(x^2-9)/(x-3)$ and writes domain $\mathbb{R}$. Correct the claim and find the range.

The expression equals x+3 only for $x \neq 3$. Domain: $\mathbb{R}$ except 3; range: $\mathbb{R}$ except 6. The graph has a hole at (3,6).

23 / Challenge

Find the domain and range of $\sqrt{(x-1)}/(x-1)$.

$x \geq 1$ and $x \neq 1$ give domain $(1,\infty)$. On that domain the rule is $1/\sqrt{(x-1)}$, always positive. Every $y>0$ occurs at $x=1+1/y^2$, so range: $(0,\infty)$.

24 / Challenge

Find the domain and range of $\sqrt{(9-x^2)}$. Justify both endpoints.

$9-x^2 \geq 0$ gives $-3 \leq x \leq 3$. Outputs are non-negative and at most 3. Values 0 and 3 occur at $x=\pm 3$ and $x=0$. All between occur, so domain [-3,3], range [0,3].